Name: .... Teacher: ....

SYDNEY TECHNICAL HIGH SCHOOL

(Est. 1911)

Year 12

Mathematics

Assessment Task 2

March 2013

Time allowed: 70 minutes

Instructions:

- Write your name and class at the top of this page.
- These questions must be handed in on the *top* of your answers
- Attempt all questions.
- All necessary working must be shown.
- Begin each question on a new page.
- Answer Section I on the Multiple Choice answer sheet provided.
- Answer Section II on the blank paper provided.

Section 1	Q6	Q7	Q8	Q9	Q10	TOTAL

Section I

Use the multiple choice answer sheet. Select the alternative A, B, C or D that best answers the question. Fill the response oval completely.

1. Given that $(x) = \frac{1}{(3x+1)^3}$, which is the correct expression for $f'(x)$?

   A. $\frac{-3}{(3x+1)^2}$ B. $\frac{-9}{(3x+1)^2}$ C. $\frac{-3}{(3x+1)^4}$ D. $\frac{-9}{(3x+1)^4}$

2. If $f'(x) < 0$ and $f''(x) > 0$ for all x over a given domain, which of the following describes the graph of $y = f(x)$?

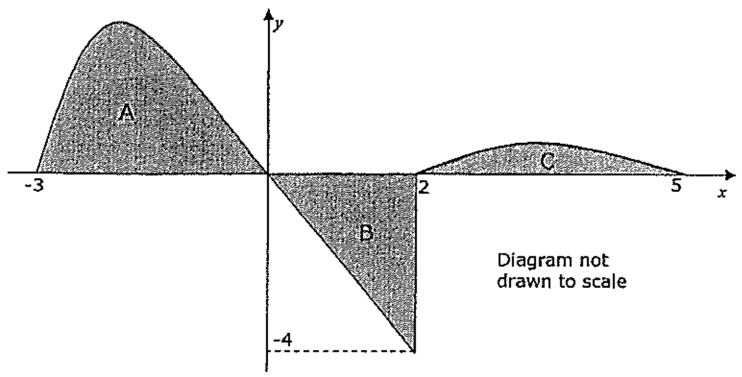
   A. Increasing and concave up

   B. Increasing and concave down

   C. Decreasing and concave up

   D. Decreasing and concave down

3. The graph of $y = f(x)$ is shown in the diagram below. The shaded areas are bounded by the curve and the x-axis. The area of region A is 8 square units and the area of region C is 1 square unit.



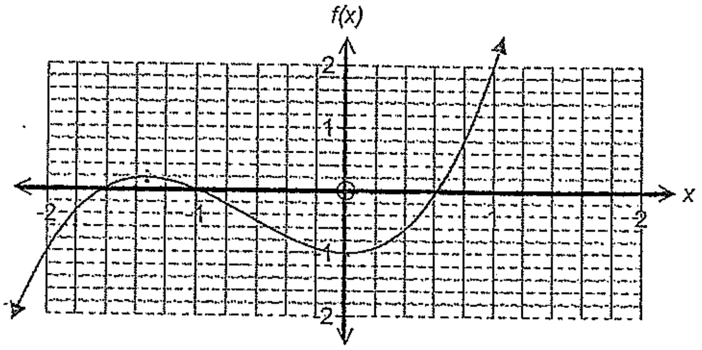
   The value of $\int_{-3}^{5} f(x)dx$ is:

   A. 5 B. 13 C. 1 D 17

4. $\sum_{n=5}^{30} (2n-1) =$

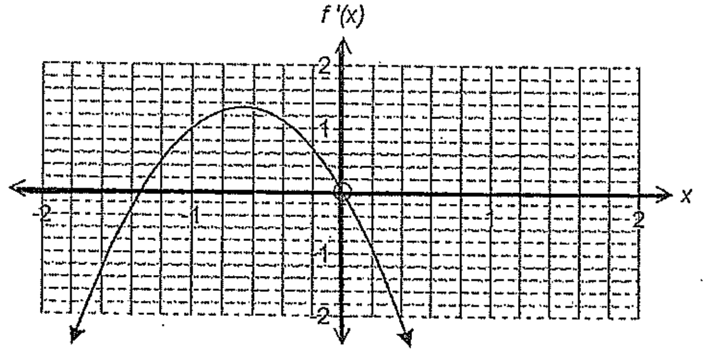
   A.59 B. 50 C. 884 D. 85

5.

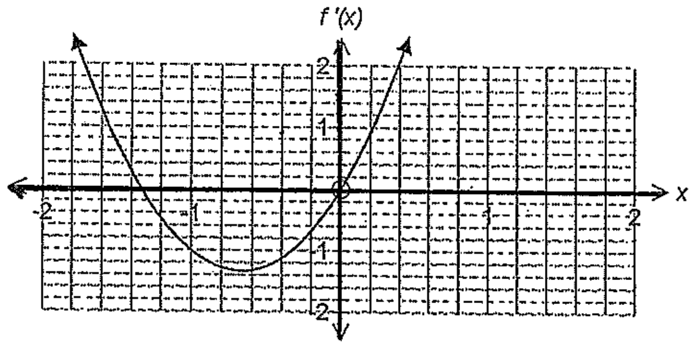


Given the *f(x)* curve above, which of the following represents the curve for *f '(x)*?

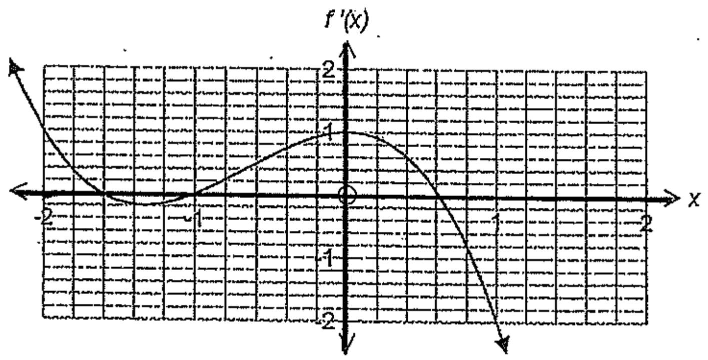
(A)



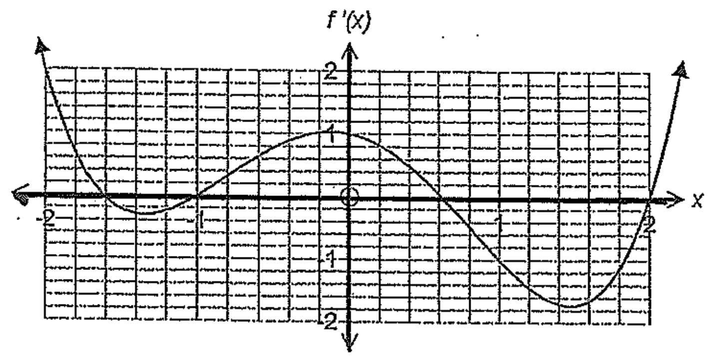
(B)



(C)



(D)



Section II

Total marks 55
Attempt questions 6 – 10
Allow about 65 minutes for this section
Show all necessary working out
Start each question on a new page

QUESTION 6 (11 marks) **MARKS**

a) Differentiate

i) $y = (x+3)(x^2-1)$ 2

ii) $y = x\sqrt{2x-5}$ 2

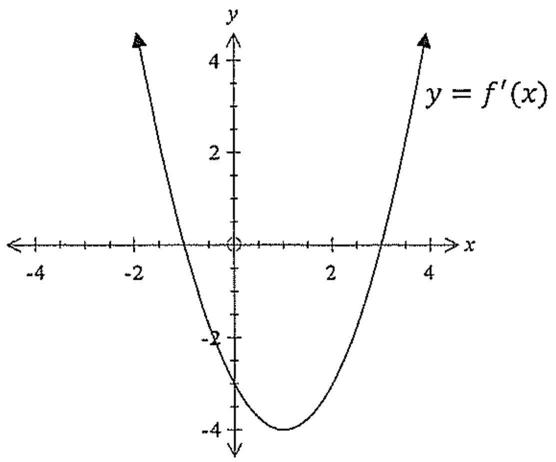
iii) $y = \frac{2x+3}{x^2+1}$ 2

b) The first term of a Geometric Series is 16 and the common ratio is $\frac{1}{n}$.

i) For what values of n will this series have a limiting sum? 2

ii) Calculate the limiting sum of the series where $n = 4$ 2

c) Below is a graph of $y = f'(x)$. Given that f(-1) = 3 and f(3) = - 1, sketch a graph of $y = f(x)$. 1



QUESTION 7 (11 marks) Start a new page **MARKS**

a) The first 3 terms of an Arithmetic Progression are 50, 43, 36. If the last term is -27, find the sum of the series. 3

b) For the curve $y = 2x^3 - 6x^2 - 18x + 1$

i) find the stationary points and determine their nature 3

ii) find the co-ordinates of any points of inflexion. 2

iii) Sketch the curve for the domain $-2 \leq x \leq 5$ 2

iv) What is the absolute maximum value of the function in this domain? 1

QUESTION 8 (11 marks) Start a new piece of paper **MARKS**

a) Find the equation of the tangent to the curve $y = 2x^2 - 2$ at the point where the tangent is parallel to the line $y = 4x + 1$ 3

b) Find

i) $\int \sqrt{x^3}\, dx$ 1

ii) $\int \frac{2x-1}{x^3}\, dx$ 2

c) Joan deposits \$350 into a special savings account on the first day of each month for two years. The interest rate is 9 %p.a. compounded monthly. Find the total amount in her savings account at the end of the two year period. 3

d) The gradient function for a curve which passes through the point (1, 2) is $4x^3 - 3x^2 + 6$. Find the equation of the curve. 2

QUESTION 9 (11 marks) Start a new piece of paper **MARKS**

a) Evaluate $\int_3^5 (3x-2)^5 dx$ 2

b) A sum of $15 000 is borrowed at 12% pa interest, calculated on the balance owing at the end of each month. The money is to be repaid at monthly intervals over 5 years.

i) If M stands for the monthly repayment, show that the amount owing at the end of the second month is given by 2
$A_2 = 15\,000\,(1.01)^2 - M\,(1.01+1)$

ii) Write a general expression for the amount owing after n months. 1

iii) Find the monthly repayment. 3

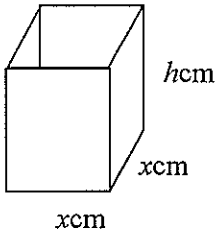
c) Find the area bounded by the curve $y = x^3 + 1$ and the x-axis between $x = -1$ and $x = 3$. 3

QUESTION 10 (11 marks) Start a new page **MARKS**

a) Find a value for n which when added to each 2, 5 & 9 will give a set of three numbers in geometric progression. 2

b) Find the area between the curve $y = \sqrt{2x-1}$, the y-axis and the lines $y = 1$ & $y = 3$ 3

c) A box with a square base and an open top is made of thin material. The box is to have a capacity of $32cm^3$



i) Find an expression for the height of the box, $h$cm, in terms of x. 1

ii) Show that the surface area, A, of the box is given by
$A = x^2 + \frac{128}{x}$ 2

iii) Find the dimensions of the box that give the minimum surface area. 3

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Name ______________________________ Teacher ______________________________

Mathematics

March 2013

SECTION I

Completely fill the response oval representing the most correct answer.

1. A ⬭ B ⬭ C ⬭ D ⬭
2. A ⬭ B ⬭ C ⬭ D ⬭
3. A ⬭ B ⬭ C ⬭ D ⬭
4. A ⬭ B ⬭ C ⬭ D ⬭
5. A ⬭ B ⬭ C ⬭ D ⬭

SECTION 1

1. D
2. C
3. A
4. C
5. B

SECTION 2

QUESTION 6

a) i) $y = x^3 + 3x^2 - x - 3$

$\frac{dy}{dx} = 3x^2 + 6x - 1$

ii) $y = x(2x-5)^{\frac{1}{2}}$

$\frac{dy}{dx} = 1.(2x-5)^{\frac{1}{2}} + \frac{1}{2}(2x-5)^{-\frac{1}{2}} \times 2 \times x$

$= \sqrt{2x-5} + \frac{x}{\sqrt{2x-5}}$

iii) $y = \frac{2x+3}{x^2+1}$

$\frac{dy}{dx} = \frac{2(x^2+1) - 2x(2x+3)}{(x^2+1)^2}$

$= \frac{2x^2+2-4x^2-6x}{(x^2+1)^2}$

$= \frac{-2x^2-6x+2}{(x^2+1)^2}$

b) $a = 16$, $r = \frac{1}{n}$

i) For limiting sum $|r| < 1$

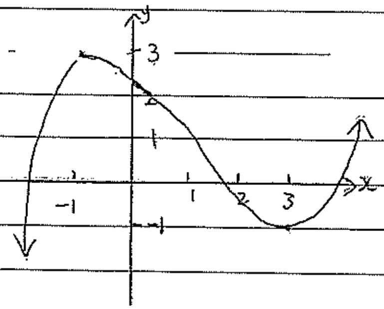
$n > 1$, $n < -1$

ii) $S = \frac{a}{1-r}$

$= \frac{16}{1-\frac{1}{4}}$

$= 21\frac{1}{3}$

c)



QUESTION 7

a) $a = 50$, $d = -7$, $T_n = -27$

$T_n = a + (n-1)d$

$-27 = 50 + (n-1) \times -7$

$-77 = -7(n-1)$

$n - 1 = 11$

$n = 12$

$S_n = \frac{n}{2}(a + l)$

$S_{12} = \frac{12}{2}(50 - 27)$

$= 138$

b) $y = 2x^3 - 6x^2 - 18x + 1$

i) $\frac{dy}{dx} = 6x^2 - 12x - 18$

$\frac{d^2y}{dx^2} = 12x - 12$

Stat pts at $\frac{dy}{dx} = 0$

$6x^2 - 12x - 18 = 0$

$6(x^2 - 2x - 3) = 0$

$(x-3)(x+1) = 0$

$x = 3$, $x = -1$

$y = -53$ $y = 11$

$\therefore$ Stat pts at $(3, -53)$ + $(-1, 11)$

at $x = 3$, $\frac{d^2y}{dx^2} > 0$ $\therefore$ concave up

$\therefore$ Min tp at $(3, -53)$

at $x = -1$, $\frac{d^2y}{dx^2} < 0$ $\therefore$ concave down

$\therefore$ Max tp at $(-1, 11)$

ii) For inflexion pts $\frac{d^2y}{dx^2} = 0$

$12x - 12 = 0$

$x = 1$

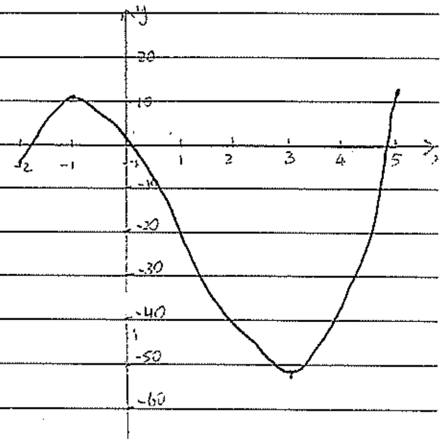
$y = -21$

x	$\frac{1}{2}$	1	$1\frac{1}{2}$
$\frac{d^2y}{dx^2}$	-	0	+

$\therefore$ Concavity changes

$\therefore$ Inflexion pt at $(1, -21)$

iii)



iv) Absolute maximum = 11

QUESTION 8

a) $y = 2x^2 - 2$

$\frac{dy}{dx} = 4x$

$m = 4x$

$y = 4x + 1$

$m_2 = 4$

$\therefore 4x = 4$ (parallel lines have the same gradient)

$x = 1$

$y = 0$

$y - y_1 = m(x - x_1)$

$y - 0 = 4(x - 1)$

$y = 4x - 4$

b) First \$350 $= 350 \times 1.0075^{24}$

2^{nd} \$350 $= 350 \times 1.0075^{23}$

$\vdots$

last \$350 $= 350 \times 1.0075$

$\text{total} = \frac{350 \times 1.0075\,(1.0075^{24} - 1)}{1.0075 - 1}$

$= \$9234.71$

c) i) $\int x^{\frac{3}{2}}\,dx = \frac{2}{5}x^{\frac{5}{2}} + c = \frac{2}{5}\sqrt{x^5} + c$

ii) $\int \frac{2x - 1}{x^3}\,dx = \int \left(\frac{2}{x^2} - \frac{1}{x^3}\right)dx$

$= \int (2x^{-2} - x^{-3})\,dx$

$= -2x^{-1} + \frac{1}{2}x^{-2} + c$

$= -\frac{2}{x} + \frac{1}{2x^2} + c$

d) $\frac{dy}{dx} = 4x^3 - 3x^2 + 6$

$y = x^4 - x^3 + 6x + c$

at $x = 1$, $y = 2$ $\quad c = -4$

$\therefore y = x^4 - x^3 + 6x - 4$

Typo

QUESTION 9

a) $\int_3^5 (3x - 2)^5\,dx$

$= \left[\frac{1}{6 \times 3}(3x - 2)^6\right]_3^5$

$= \frac{1}{18}(3 \times 5 - 2)^6 - \frac{1}{18}(3 \times 3 - 2)^6$

$= 261\,620$

b) \$15 000 $\quad r = 1\%$ per month $\quad n = 60$

i) $A_1 = \$15000 \times 1.01 - M$

$A_2 = (15000 \times 1.01 - M) \times 1.01 - M$

$= 15\,000 \times 1.01^2 - 1.01M - M$

$= 15\,000 \times 1.01^2 - M(1.01 + 1)$

ii) $A_n = 15\,000 \times 1.01^n - M(1 + 1.01 + \cdots + 1.01^{n-1})$

iii) $A_{60} = 15000 \times 1.01^{60} - M(1 + 1.01 + \ldots + 1.01^{59})$

But $A_{60} = 0$

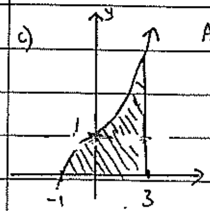
$15000 \times 1.01^{60} - M(1 + 1.01 + \cdots + 1.01^{59}) = 0$

$M = \frac{15\,000 \times 1.01^{60}}{1 + 1.01 + \ldots + 1.01^{59}}$

$= \frac{15\,000 \times 1.01^{60}}{\frac{1\,(1.01^{60} - 1)}{1.01 - 1}}$

$= \$333.67$

c)



$A = \int_{-1}^{3} (x^3 + 1)\,dx$

$= \left[\frac{1}{4}x^4 + x\right]_{-1}^{3}$

$= \frac{1}{4}(3)^4 + 3 - \left(\frac{1}{4}(-1)^4 + (-1)\right)$

$= \frac{93}{4} + \frac{3}{4}$

$= 24 \text{ units}^2$

Typo

QUESTION 10

a) $2+n$, $5+n$, $9+n$

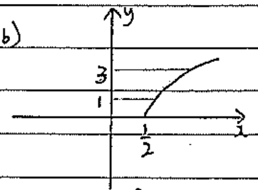
$$\frac{5+n}{2+n} = \frac{9+n}{5+n}$$

$$(5+n)^2 = (9+n)(2+n)$$

$$25+10n+n^2 = 18+11n+n^2$$

$$n = 7$$

b)



$$y = \sqrt{2x-1}$$

$$y^2 = 2x-1$$

$$y^2+1 = 2x$$

$$x = \tfrac{1}{2}(y^2+1)$$

$$\text{Area} = \int_1^3 \tfrac{1}{2}(y^2+1)\,dy$$

$$= \tfrac{1}{2}\left[\tfrac{1}{3}y^3+y\right]_1^3$$

$$= \tfrac{1}{2}\left(\tfrac{1}{3}(3)^3+3-(\tfrac{1}{3}+1)\right)$$

$$= 5\tfrac{1}{3} \text{ units}^2$$

c) i) $V = x \times x \times h$

$$32 = x^2h$$

$$h = \frac{32}{x^2}$$

ii) $A = x^2 + 4xh$

$$= x^2 + 4x \times \frac{32}{x^2}$$

$$= x^2 + \frac{128}{x} \qquad = x^2 + 128x^{-1}$$

iii) $\frac{dA}{dx} = 2x - 128x^{-2}$

$$\frac{d^2A}{dx^2} = 2 + 256x^{-3}$$

stat pts at $\frac{dA}{dx} = 0$

$$2x - 128x^{-2} = 0$$

$$2x - \frac{128}{x^2} = 0$$

$$\frac{2x^3-128}{x^2} = 0$$

$$2(x^3-64) = 0$$

$$(x-4)(x^2+4x+16) = 0$$

$$x = 4$$

$$h = 2$$

at $x=4$, $\frac{d^2A}{dx^2} > 0$ ∴ concave up

∴ Minimum at $x = 4$

∴ Dimensions are $4\text{cm} \times 4\text{cm} \times 2\text{cm}$